Postulates of Quantum Mechanics

The six rules that define how the quantum world works

A self-study guide for first-year engineering students

Why do we need postulates?

CLASSICAL PHYSICS

- A particle has a definite position and momentum.
- Measuring does not disturb it.
- Same start gives same result, always.

QUANTUM MECHANICS

- A system is described by a state, not a trajectory.
- Measurement changes the state.
- Same start can give different results, with fixed probabilities.

THE BIG IDEA

A postulate is a basic rule we accept because every experiment so far agrees with it (double-slit, photoelectric effect, atomic spectra). We do not derive postulates; we use them to derive everything else. Roadmap: State, Observables, Measurement values, Probability, Collapse, Time evolution.

The wavefunction Ψ(x, t)

WHAT IS IT?

A mathematical function that describes the quantum state of a particle. It is the quantum replacement for "position and velocity". Its link to matter waves: a free particle of momentum p has wavelength λ = h/p.

$$\text{Free particle: } \Psi(x,t) = A\, e^{i(kx - \omega t)}$$
$$k = p/\hbar, \quad \omega = E/\hbar$$

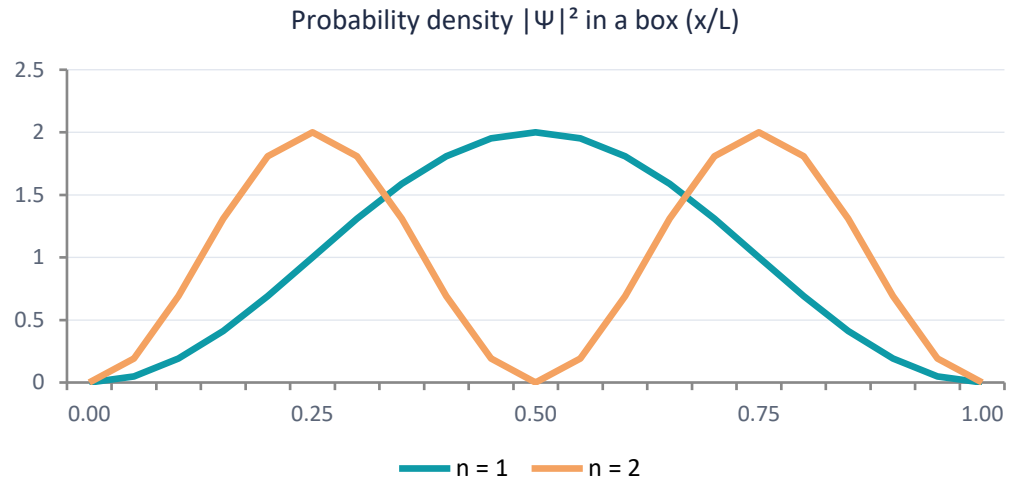
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Electron at 100 V: λ ≈ 0.12 nm, similar to atomic spacing. That is why electrons diffract from crystals.

PROPERTIES OF Ψ

- Complex-valued: Ψ = a + ib
- Not directly measurable
- Contains all information about the state
- Obeys superposition
- Evolves by the Schrödinger equation

Physical significance: $|\Psi|^2$ (Born interpretation)



$$P(a \le x \le b) = \int_a^b |\Psi|^2 \, dx \qquad \int |\Psi|^2 \, dx = 1$$

$|\Psi|^2$ is a probability density: probability per unit length. Peaks show where the particle is most likely to be found. For n = 2 the particle is never found at the centre.

A VALID Ψ MUST BE

- Single-valued
- Finite everywhere
- Continuous
- Smooth (continuous slope)
- Square-integrable, so it can be normalized

Postulate 1

The State of a System

WHAT IT SAYS

Everything that can be known about an isolated system is contained in a state vector (wavefunction) Ψ. It lives in a complex vector space called Hilbert space.

$$\Psi(x,t) \quad \text{with} \quad \int |\Psi|^2 \, dx = 1$$

WHY IT MATTERS

Ψ replaces the classical idea of "position and velocity". The normalization condition says the particle must be found somewhere.

EXAMPLE / PICTURE IT

An electron in an atom is not a tiny ball on an orbit. It is described by a wavefunction spread over space; $|\Psi|^2$ is large where the electron is likely to be found.

States can be added (superposition): if Ψ_1 and Ψ_2 are allowed states, so is $a\Psi_1 + b\Psi_2$.

Postulate 2

Observables are Operators

WHAT IT SAYS

Every measurable quantity (observable) corresponds to a Hermitian operator acting on the state.

$$\hat{x} = x \qquad \hat{p} = -i\hbar\,\partial/\partial x$$
$$\hat{H} = \hat{p}^2/2m + V(x)$$

WHY IT MATTERS

Hermitian operators have real eigenvalues, so measured values are always real numbers, as they must be.

EXAMPLE / PICTURE IT

Position: multiply Ψ by x.
Momentum: differentiate Ψ with respect to x and multiply by −iℏ.
Energy: the Hamiltonian $\hat{H}$ (kinetic + potential).

Order matters: $\hat{x}\hat{p} \neq \hat{p}\hat{x}$. This leads to the uncertainty principle.

Postulate 3

Measurement Gives Eigenvalues

WHAT IT SAYS

The only possible results of measuring an observable A are the eigenvalues a of its operator.

$$\hat{A}\,\psi_a = a\,\psi_a$$

WHY IT MATTERS

Quantities like energy in an atom come in discrete allowed values. That is why atoms emit light only at specific wavelengths.

EXAMPLE / PICTURE IT

Particle in a box of width L:

$\hat{H}\,\psi_n = E_n\,\psi_n$

$E_n = n^2\pi^2\hbar^2 / (2mL^2)$, $n = 1, 2, 3\ldots$

You will never measure an energy between E_1 and E_2.

Postulate 4

Probability: the Born Rule

WHAT IT SAYS

If the state is a combination of eigenstates, $\Psi = \Sigma\, c_n \psi_n$, the probability of getting eigenvalue a_n is $|c_n|^2$. For position, $|\Psi(x)|^2\, dx$ is the probability of finding the particle near x.

$$P_n = |c_n|^2 \qquad \langle A \rangle = \int \Psi^* \hat{A}\, \Psi \, dx$$

WHY IT MATTERS

Quantum theory predicts probabilities, not single outcomes. ⟨A⟩ is the average over many identical experiments.

EXAMPLE / PICTURE IT

State $\Psi = 0.6\, \psi_1 + 0.8\, \psi_2$
$P(E_1) = 0.36$
$P(E_2) = 0.64$
Total = 1 ✓

Repeat the experiment 1000 times: about 360 results give E_1 and 640 give E_2.

Postulate 5

Collapse of the State

WHAT IT SAYS

Right after a measurement gives the value a_n, the system is in the eigenstate ψ_n. Measuring again immediately gives a_n with certainty.

$$\Psi = \Sigma\, c_n \psi_n \xrightarrow{\text{measure}} \psi_n$$

WHY IT MATTERS

Measurement is not passive. It changes the state, from a superposition to one definite eigenstate.

EXAMPLE / PICTURE IT

An electron is in a superposition of "spin up" and "spin down". You measure and get "up". The state is now "up".

Measure again: you get "up" every time. The earlier superposition is gone.

Postulate 6

Time Evolution

WHAT IT SAYS

Between measurements, the state changes smoothly and predictably according to the Schrödinger equation.

$$i\hbar \, \partial\Psi/\partial t = \hat{H}\,\Psi$$

WHY IT MATTERS

Evolution is deterministic: given Ψ now, Ψ later is fixed. Randomness enters only at measurement.

EXAMPLE / PICTURE IT

For an energy eigenstate:
$\Psi_n(x,t) = \psi_n(x)\, e^{(-iE_nt/\hbar)}$

Only the phase rotates, so $|\Psi|^2$ does not change. This is called a stationary state.

Worked example: putting it together

1 State

$\Psi = (1/\sqrt{3})\,\psi_1 + \sqrt{(2/3)}\,\psi_2$. Check: $1/3 + 2/3 = 1$, so it is normalized.

2 Operator

Observable is energy, so use $\hat{H}$ with $\hat{H}\psi_n = E_n\psi_n$.

3 Possible results

Only E_1 or E_2 (eigenvalues).

4 Probabilities

$P(E_1) = 1/3 \approx 33\%$, $P(E_2) = 2/3 \approx 67\%$.

5 Average

$\langle E \rangle = (1/3)E_1 + (2/3)E_2$.

6 After measuring

If you got E_2, the state collapses to ψ_2.

Summary: all six at a glance

#	Postulate	Key formula	One-line idea
1	State	Ψ, $\int \lvert\Psi\rvert^2 dx = 1$	System described by a wavefunction
2	Observables	$\hat{p} = -i\hbar\, \partial/\partial x$	Measurable quantity = Hermitian operator
3	Measured values	$\hat{A}\psi = a\psi$	Results are eigenvalues
4	Probability	$P = \lvert c\rvert^2$	Born rule gives chances
5	Collapse	$\Psi \rightarrow \psi_n$	Measurement changes the state
6	Evolution	$i\hbar\, \partial\Psi/\partial t = \hat{H}\Psi$	Smooth and deterministic between measurements

Common misconceptions

MYTH: "Measurement means a human is watching"

FACT: Any interaction that records the value counts as a measurement.

MYTH: "Ψ is a real wave in space"

FACT: Ψ is a complex amplitude; only $|\Psi|^2$ has a direct physical meaning.

MYTH: "Quantum means everything is random"

FACT: Evolution is deterministic. Only individual measurement results are probabilistic.

MYTH: "Superposition means being in two places at once"

FACT: It means the state is a sum of possible states, each with an amplitude.

Test yourself

TRY THESE (answers on the next slide)

1. Normalize $\Psi = A \cdot x(L-x)$ on $0 \le x \le L$, or explain why normalization is needed.
2. State $\Psi = 0.8\,\psi_1 + 0.6\,\psi_2$. Find $P(E_1)$ and $P(E_2)$.
3. Why must observables be Hermitian operators?
4. A measurement gave E_2. What is the state immediately afterwards?
5. Which postulate is deterministic and which is probabilistic?

Answers and key takeaways

ANSWERS

1. $A = \sqrt{(30/L^5)}$
2. $P(E_1) = 0.64$, $P(E_2) = 0.36$
3. Hermitian means real eigenvalues, so measured values are real.
4. Eigenstate ψ_2 (collapse).
5. Evolution: deterministic. Measurement: probabilistic.

REMEMBER

- State = wavefunction Ψ
- Observable = operator
- Result = eigenvalue
- Chance = $|\text{amplitude}|^2$
- Measurement = collapse
- Between measurements = Schrödinger equation
- Further reading: Griffiths, Introduction to Quantum Mechanics (Ch. 1–3)